# Geometry seminar, Yonsei

## §1 Motivation

- Lurie-Pridham $\rightsquigarrow$ this is Lie-Com
- nc Lurie-Pridham $\rightsquigarrow$ this is Ass-Ass

## §2 Conilpotent KD (after Quillen, Hinich, L-H, Positselski)

- $\exists$ a Quillen eq $B : dgAlg^{aug} \longleftrightarrow dgCog^{conil} : \Omega$
- FMP $\Leftrightarrow$ ~~the~~ pro representability:

$$F(R) \simeq Map_{pcdgAlg}(A_F, R)$$

$$\simeq Map_{dgCog^{conil}}(R^*, A_F^*) \qquad \cancel{X_F^*} = A_F^*$$

$$\simeq Map_{dgAlg^{aug}}(\Omega(R^*), \cancel{\Omega(A_F^*)}\, B_F) \qquad B_F = \Omega(A_F^*)$$

so $B_F \longmapsto Map(\cancel{\Omega(-)^*}\, \Omega(-)^*, B_F)$ is an equiv
$dgAlg \to FMP$.

- $\underline{\text{Conilpotency}} \longleftrightarrow$ FMPs accept $\underline{\text{Artinian local}}$ dg algs as inputs

$\boxed{Q}$ How can we drop conilpotency?

Intuitively: corresponds to FMPs ~~with~~ taking all fin dim algs as input

Commutatively: less interesting; split as $\times$s of locals

Noncommutatively: $\underline{\text{interesting}}$! $M_2(k)$

So we want to drop the co/augmentations too.

this corresponds to <u>curvature</u>

§3 | <u>Curvature</u> a cuAlg has $d^2 = [h,-]$ & $dh = 0$

Morphisms have two cpts:

$f: A \to B$ grAlgs 'uncurved'

$b \in B^1$ $f(da) = d(fa) + [b, fa]$ 'change of curvature'

$f(h_A) = h_B + db + b^2$

an <u>MC element</u> is $x \in A^1$ with $h + dx + x^2 = 0$

we have $MC(A) \simeq Hom(k, A)$

<u>Prop</u> ~~A~~ $MC(A) \neq \emptyset \Leftrightarrow A$ is cu/iso to a dg Alg

<u>Proof</u> if $x$ is MC, $d^x = d + [x,-]$ is a diff'l

~~$(A, d^x)$~~ ~~iso~~ $(id, x): (A, d^x) \xrightarrow{\simeq} (A, d)$

$\Leftarrow$ $0 \in MC(dgAlg)$

~~Prop~~ Cor $dgAlg \simeq cuAlg_{k/}$

~~Similarly~~ ~~dg~~ Similarly, $dgCog \simeq cuCog_{/k}$

~~§4~~. There's an <u>extended bar construction</u>

$\check{B}: cuAlg \leftrightarrow cuCog : \Omega$ (Anel-~~J~~oyal ~~§~~, Guan-Lazarev)

which slices to adjunctions

$dgAlg \rightleftarrows cuCog^{coaug}$

$cuAlg^{aug} \rightleftarrows dgCog$

$dgAlg^{aug} \rightleftarrows dgCog^{coaug}$

Thm $\exists$ model str on cuAlg & cuCog

making $\Omega \dashv \check{B}$ into a Quillen equiv.

~~These~~ also slice to Quillen equiv. models.

what are the WEs? no notion of qiso.

if $E$ is a dgAlg, dg cat $MC_{dg} E \subseteq \mathrm{Tw} E$ $(-1)^{\deg e} e$

~~elts are~~ objs MC elts

~~Hom~~ maps 2-sided twists $de + ye - \tilde{e}x$

(might be empty!)

$\mathrm{Hom}(C,A)$ cuAlg.

$A \to A'$ is an MC equiv if $\forall C$

$MC_{dg}\mathrm{Hom}(C,A) \longrightarrow MC_{dg}\mathrm{Hom}(C,A')$ quasi-equiv.

[Enough to check on $C$ fin dim & on isoclasses in $H^0$]

Similar def for cuCogs

Consequence representability results for MC stacks

(~~[illegible]~~ certain $\infty$-functors $(cu)\mathrm{Alg}^{fd} \to s\mathrm{Set}$)

or noncommutative moduli spaces

$\left( (cu)\mathrm{Alg}^{fd} \longrightarrow dg\mathrm{Cat} \right)$